Note: all statements require proofs. You can make references to standard theorems from the course; however, you must state the relevant part of the theorem in your own words, unless it is a well-known named theorem. For example, "by the monotone convergence theorem," or, "we showed in lecture that the integrals of an increasing sequence of positive functions converge to the integral of their limit," are good references but, "by a convergence theorem the integrals converge," is **not** a good reference.

1. Let $(X, \mathcal{M}, \mu)$ be a finite measure space and suppose $(f_n)_{n\in\mathbb{N}}$ is a sequence of $\mathcal{M}$-measurable functions converging μ-almost everywhere to some function f. Show that $f_n \to f$ in measure.

2. Let $(X, \mathcal{M}, \mu)$ be a measure space. Fix $f \in L^\infty(X, \mu)$ and $1 \leq p < \infty$.
   (a) Show that $fg \in L^p(X, \mu)$ for all $g \in L^p(X, \mu)$.
   (b) Show that if μ is semifinite then
   $$\|f\|_\infty = \sup\{\|fg\|_p \colon g \in L^p(X, \mu),\ \|g\|_p = 1\}.$$

3. Let $f \colon \mathbb{R} \to \mathbb{C}$ be bounded and uniformly continuous, and let $g \in L^1(\mathbb{R}, m)$. For $x \in \mathbb{R}$, define
   $$h(x) := \int_{\mathbb{R}} f(x-y)g(y)\ dm(y).$$
   (a) Show that h is a bounded and uniformly continuous function on $\mathbb{R}$.
   (b) Suppose $\lim\limits_{x\to\pm\infty} f(x) = 0$. Show that $\lim\limits_{x\to\pm\infty} h(x) = 0$.

4. Suppose $F \colon [0,1] \to \mathbb{C}$ and $L > 0$ satisfy $|F(x) - F(y)| \leq L|x-y|$ for all $x, y \in [0,1]$.
   (a) Show that F is absolutely continuous on $[0,1]$.
   (b) Let μ_F be the regular complex Borel measure on $[0,1]$ satisfying $\mu_F((a,b]) = F(b) - F(a)$. (You do **not** need to show this measure exists.) Show that $|\mu_F(E)| \leq Lm(E)$ for all Borel sets $E \subset [0,1]$.

5. Suppose $F, G \colon [a,b] \to \mathbb{C}$ are absolutely continuous functions and that $G(x) \neq 0$ for all $x \in [a,b]$. Show that the quotient $\frac{F}{G}$ is absolutely continuous.